Modeling of Nonlinear Active Circuits Using FDTD Approach

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Abstract: -This paper, describes a voltage- source formulation of the extended finite-difference time-domain algorithm for the purpose of modeling nonlinear microwave devices. Based on this approach, the device-wave interaction is characterized and incorporated into FDTD algorithm. Analysis of nonlinear properties, including harmonic generation and intermodulation, can be accomplished by using a large signal device circuit model. This technique is applied to the analysis of a typical nonlinear microwave amplifier, which includes a three-terminal active MESFET device. Simulation results are in good agreement with MICROWAVE OFFICE results.

Keywords: FDTD, MESFET, Large Signal, Microwave Amplifier, Intermodulation

1 Introduction

The finite-difference time-domain (FDTD) approach was invented by K.S Yee in 1966 [1], when he developed the so called Yee's cell. This computational cell naturally interleaves electric and magnetic grid points (grid points where the electric and magnetic fields are evaluated) so that the rotational of the electric or magnetic fields can be expressed in a straightforward manner using finite difference approximations. This method remained in the shadow up until the late 80's when the simulation of open structures was made possible using absorbing boundary conditions (ABC). The Mur's ABC [2] and the Perfectly Matched Layer [3] where a big breakthrough that allowed the FDTD method to be used in a variety of problems. It was widely used to calculate microstrip discontinuities efficiently and some planar microstrip circuits, such as antennas, filters and couplers.

Recently, more attentions have been paid to the analyzing of microstrip circuits including active nonlinear devices. Some papers have been published in this area [4] [5]. In these papers, the active nonlinear devices, acting as exciting sources, were modeled by distributing equivalent circuits and incorporated into FDTD cells. This approach has its limitations, for example, the implementation of this approach will become impractical when a multi-port lumped circuit is involved in microstrip circuits. This paper presents the application of the extended FDTD method to the analysis of nonlinear phenomena in microwave circuits. A microwave amplifier with a large signal model for MESFET is used as the platform.

2 Device Model in FDTD Analysis

An active device in a microwave circuit is typically very small in size compared to a wavelength, and it can be modeled by its equivalent lumped circuit with a very high degree of accuracy. In order to include the equivalent circuit model into the FDTD analysis, the active device can be replaced by equivalent voltage sources in the active region, which introduced by Kuo *et.al* [6] [7] [9], if the voltage sources satisfy the voltage-current relationship at the input/output ports and the scattering properties of the active device.

Fig.1 illustrates the Thevenin equivalent circuits "looking into" the FDTD space lattice from the two-terminal port of a circuit device. We assume that the port is located in free space in direction of $\mathrm{E}_x, \mathrm{E}_y$ or E_z component in a cubic cell FDTD lattice. Fig.2 shows the placement of the voltage sources at one port of the active device; each source is aligned to the FDTD grid edges beneath and perpendicular to the microstrip line with each ends connected to the microstrip line or a grounded via. The active device distributed along the width only, with thickness of one cell. The vias provided between the device and ground plane may introduce some inductance in the circuit. They provide a voltage reference to the voltage sources. From the Thevenin equivalent circuit standpoint, the equivalent voltage obtained from the loop integration of the E-field along the cell boundary other than the edge of the voltage source.

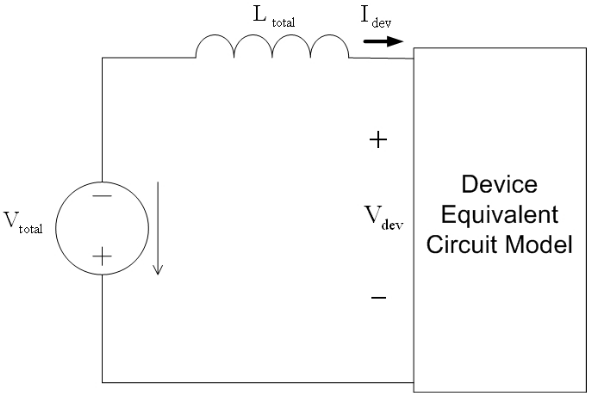

Fig.1 The Thevenin-equivalent circuit governing the device–wave inter-action

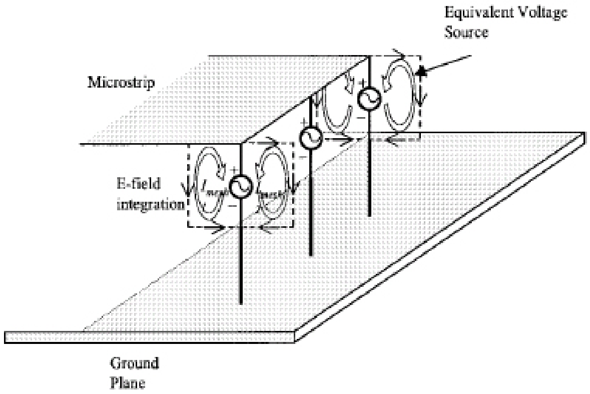

Fig.2 Placements of Equivalent-voltage sources for microstrip circuits

Two FDTD meshes contribute to the current flowing through the voltage source. For the left mesh, the integral form of Faraday's law can be expressed in terms of circuit quantities as

$$L_1 \frac{dI_{mesh,1}}{dt} = V_{loop,1} + V_{dev} \tag{1}$$

A similar formulation can be derived for the right mesh to obtain a circuit equation for the loop current which flows along the right-hand side of the active sheet.

The device current flowing into voltage source equals the sum of the two component currents in sides, right and left, of active sheet.

There are K voltage sources placed across the entire wide of microstrip line. The device current I_{dev} is the sum of all the loop currents and $\mathrm{L_i}$ is the space inductance of each mesh.

$$-L_{total} \frac{dI_{dev}}{dt} = V_{total} + V_{dev} \tag{2}$$

$$V_{total} = \sum_{i}^{2K} V_{loop,i} L_{total} \left(\frac{1}{L_i} \right) \tag{3}$$

$$L_{total} = \frac{1}{\sum_{i}^{2K} \frac{1}{L_i}} \tag{4}$$

In each time advance, the device voltage is evaluated from the state equation of the circuit in Fig.1 and subsequently used to update the electromagnetic field in the equivalent-source region. In voltage source approach the equivalent sources are related to the equivalent circuit of the device by the state equations derived from Kirchoff's current and voltage laws. The state equations are first-order differential equations expressed in a matrix form as

$$A(X)\frac{dX}{dt} = B(X)\,X + C \tag{5}$$

Where X is the state variable matrix and A, B and C are coefficient matrices. This matrix equation can be expressed as a time-stepping matrix equation by employing a forward or backward differencing scheme as

$$\left[\frac{A(X^{n+1})}{\Delta t} - B(X^{n+1}) \right] X^{n+1} - \frac{A(X^{n+1})}{\Delta t} X^{n} - C(X^{n+1}) = 0 \tag{6}$$

For linear devices, the elements of the coefficient matrices are constant and can be calculated before the time stepping begins. For nonlinear devices, the elements are varied with each time step and a Newton–Raphson method can be applied to solve this nonlinear matrix equation. Once the matrix equation (6) is solved, the terminal voltages at each port are known and can be used as the feedback to update electric-field components. The criterion of numerical stability is not only to satisfy the Courant condition in the FDTD algorithm but also to choose time step such that Jacobian matrix in Newton–Raphson root finding method is not singular. Typically the chosen Δt for the former criterion is the order of pico seconds for microwave circuits, much smaller than that for the latter as the order of nano-second, so it causes no numerical burden because of choosing a smaller Δt.

3 Analysis of Amplifier with Nonlinear Device Model

The system under consideration, as shown in Fig.3, is an amplifier. We have selected a nonlinear microwave FET for this amplifier [8]. The nonlinear equivalent circuit for this FET is shown in the Fig.4. This circuit model contains two nonlinear elements, the gate–source capacitor and the drain current.

Governed by the *PN*-junction capacitance model, the gate–source capacitor is expressed as

$$C_{gs}(v_g) = \frac{C_{gs0}}{\sqrt{1 - \frac{v_g}{\phi_{bi}}}} \tag{7}$$

$C_{gs0} = 3\,pF$, $\phi_{bi} = 0.7$

And drain current

$$I_{ds}(v_g, v_d) = \left(A_0 + A_1 v_{G'S'} + A_2 v_{G'S'}^2 + A_3 v_{G'S'}^3\right) tanh(\alpha v_d) \tag{8}$$

$A_0 = 0.5304,\ A_1 = 0.2595, A_2 = -0.05402,\ A_3 = -0.0305$
$\alpha = 0.1$

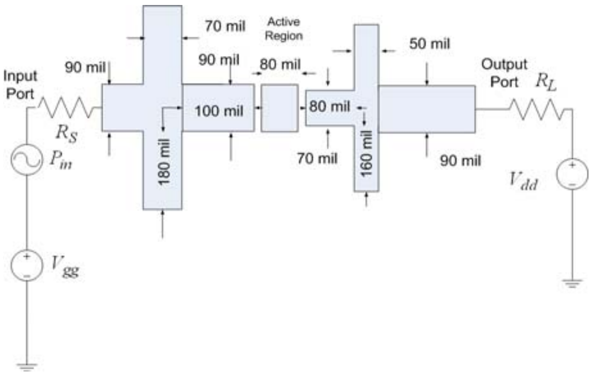

Fig.3 The structure and the dimension of a microwave amplifier.

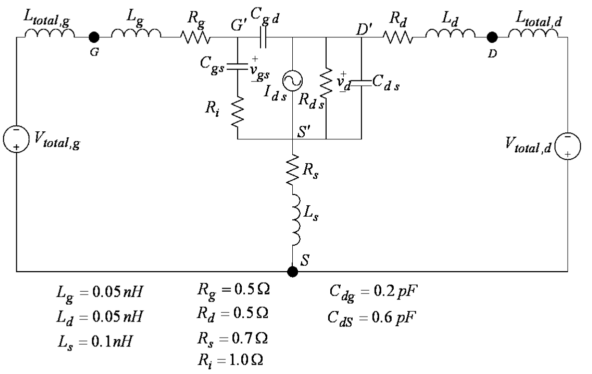

Fig.4 The large-signal model of a MESFET used in this paper

State variables are chosen as

$$X = \left\lfloor v_g \quad v_{G'D'} \quad v_d \quad i_{Lg} \quad i_{Ld} \right\rfloor \tag{9}$$

The amplifier excludes biasing circuits, so dc biasing is established by directly applying dc sources, V_{gg} and V_{dd}, each with a source impedance of $50\,\Omega$, at the input/output ports. The biasing conditions chosen are $V_{GS} = -0.81\,V$ and $V_{DS} = 6.4\,V$. The circuit is designed to match at 6 GHz. The size of the MESFET, which resides in the region of 80 mil in the longitudinal direction, is much smaller than the guided wavelength at 6 GHz. Substrate thickness of 31mil with a dielectric constant of 2.33 is selected for this simulation. The entire computation domain is divided into 65 100 18, cells with cell size 10mil, 10mil and 7.75mm. Brenger perfectly match layer (PML) [3] is applied on the truncated boundary to absorb out-going waves. The metal is assumed to be a perfect conductor of zero thickness. We have used ten and eight voltage sources, along the microstrip width in gate and drain ports. The method is first applied to examine the small-signal response of the circuit. A Gaussian pulse modulated at 6 GHz is used as the ac signal and the amplitude is small to allow the circuit to operate in the linear region. A Gaussian pulse modulated at 6 GHz is used as the ac signal and the amplitude is small to allow the circuit to operate in the linear region. FDTD simulation starts with dc excitation by using an exponential rising function to reduce the transient time, and then an ac signal is imposed upon the input port. The formulation of incorporating a voltage source with source impedance into the FDTD algorithm is described in [5].

In this simulation, it takes 20000 time steps and the execution time takes about 1.5 hour using the AMD SEMPRON™ 2600+1.8 GHZ with 512 RAM.

A circuit simulator, MCRWAVE OFFICE 2002, is also applied to simulate the circuit. Particularly, the vias at the source port are considered by adding an inductor of 0.05 nH, a typical inductance, connecting the source port to the ground nodes.

In frequency domain simulation, the amplifier provides a gain of 9.77 dB at 6.0 GHz and input matching point occurs at 6 GHz. This data is used for comparison. The results based on FDTD and frequency-domain circuit analysis are plotted from 3.0 to 10.0 GHz in Fig.5. With voltage source approach the matching dip occurs at 6.15 GHz and again of 9dB at 6.0 GHz. This deviation may come from the modeling of the vias at the source port.

Time-domain response of amplifier is shown in Fig.6. A large-signal analysis to evaluate the nonlinear phenomena has been carried out by computing the power delivered to the load as

$$P_{out}(\omega) = V_L^2(\omega) / R_L \qquad (10)$$

The input signal for this analysis is a single tone frequency at 6.0 GHz and the frequency domain information is obtained from a Fourier analysis of the time-domain signal. Fig.7 shows the harmonics at the output. Note that the output power appears at harmonic frequencies only.

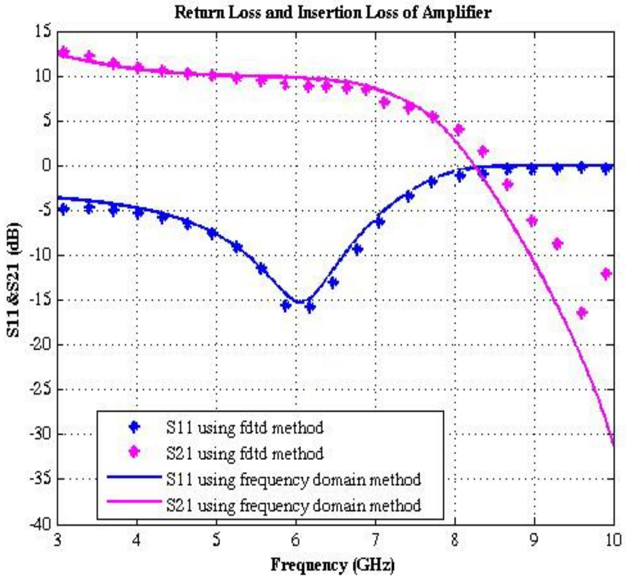

Fig.5 Comparison between the FDTD with equivalent voltage source analysis and the frequency-domain

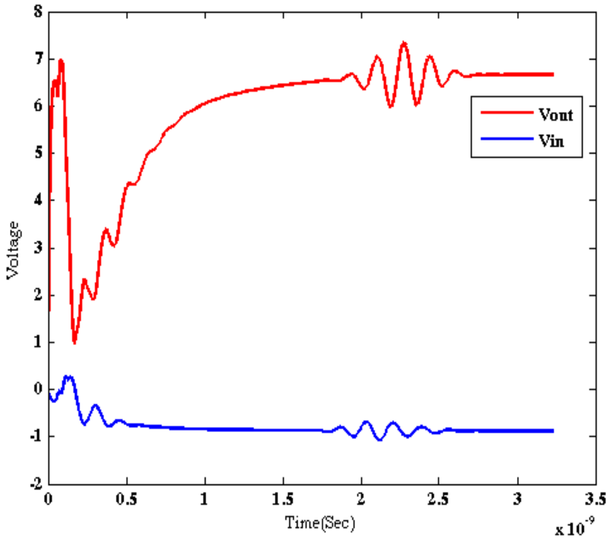

Fig.6 Time-domain response of amplifier

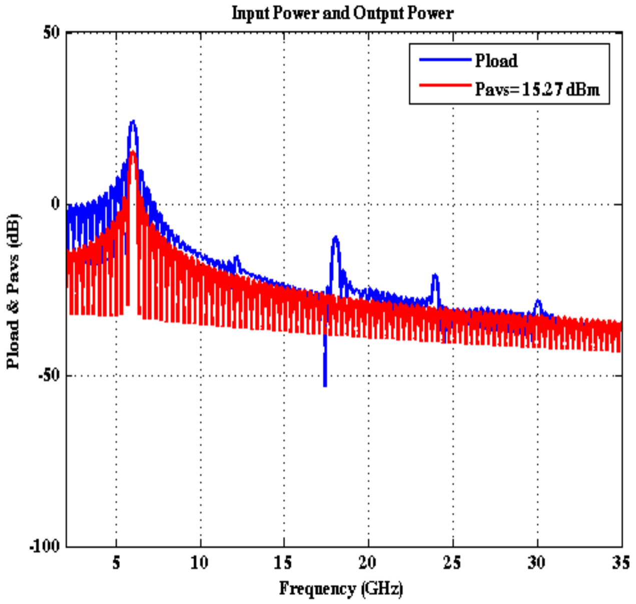

Fig.7 The spectrum of the output power using single-tone excitation at 6 GHz.

The power between harmonics is actually the numerical noise from the Fourier transform. In Fig.8 the output power to the input power is shown.

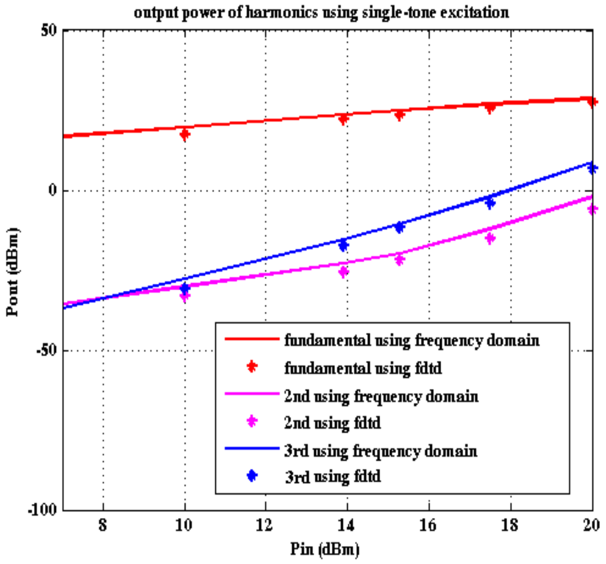

Fig.8 The output power of amplifier using single-tone excitation with different power

A complete characterization of the amplifier also requires a two-tone excitation test to verify intermodulation. This event is inspected by two tone signal at 3 and 6 GHz of the same input power level. Result is shown in Fig.9. The output power appears only at the frequency of mixing frequencies, or intermodulation products, which arise as linear combinations of 3 and 6 GHz.

These results of nonlinear analyses by FDTD simulation are in good agreement with those obtained by MICROWAVE OFFICE 2002 simulation. The analysis of these system responses shows the capability of the extended FDTD method in dealing with nonlinear active microwave circuits.

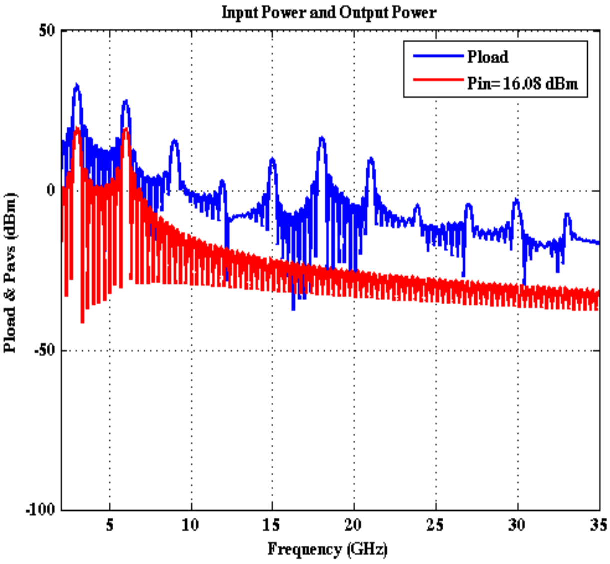

Fig.9 The spectrum of the output power using two tones excitation at 3 and 6 GHz with the same power.

4 Conclusion

Thevenin equivalent approach for the modeling of nonlinear active microwave circuits has been studied. With use of voltage equivalent sources, the FDTD method has been extended to include three terminal nonlinear active microwave devices and analyze the entire microwave circuits. This approach maintains the features of full-wave analysis and performs accurate electromagnetic field simulation of microwave and millimeter wave circuits. Although full-wave simulators are still much more time-consuming as compared to circuit simulators, this analysis becomes necessary and provides useful information for circuit design in the environment where electromagnetic effect of radiation and coupling effect must be considered.

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